Introduction to Deep Learning

4. Linear Regression, Basic Optimization

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Alex Smola and Mu Li

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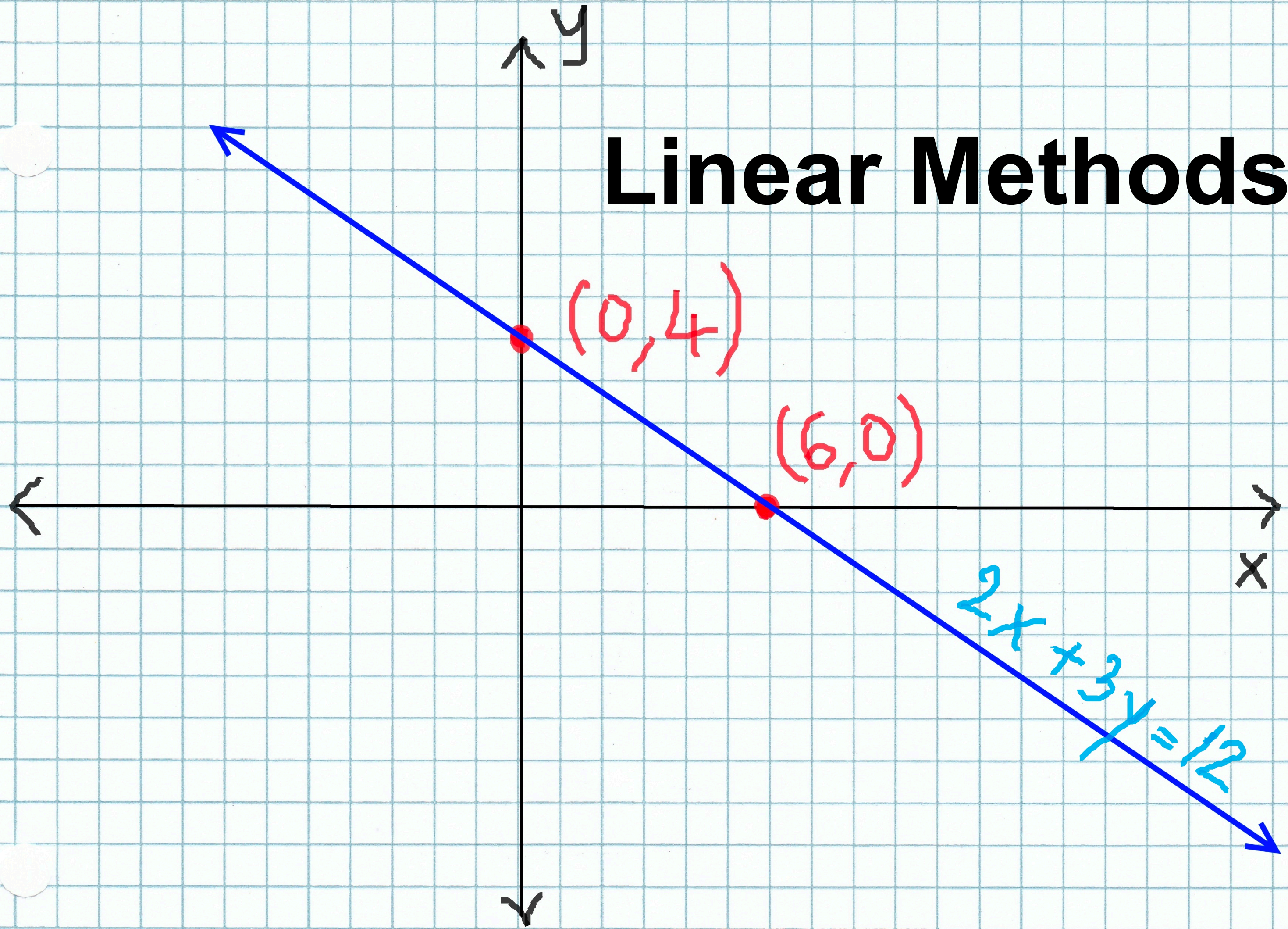
Linear Methods



House Buying 101

- Pick a house, take a tour, and read facts
- Estimate its price, bid

Listing price from agent

$5,498,000	7	5	4,865 Sq. Ft.
Price	Beds	Baths	$1130 / Sq. Ft.

Redfin Estimate: $5,390,037 On Redfin: 15 days

Predicted sale price

Virtual Tour
- Branded Virtual Tour
- Virtual Tour (External Link)

Parking Information
- Garage (Minimum): 2
- Garage (Maximum): 2
- Parking Description: Attached Garage, On Street
- Garage Spaces: 2

Multi-Unit Information
- # of Stories: 2

School Information
- Elementary School: El Carmelo El
- Elementary School District: Palo
- Middle School: Jane Lathrop Stan
- High School: Palo Alto High
- High School District: Palo Alto Un

Interior Features

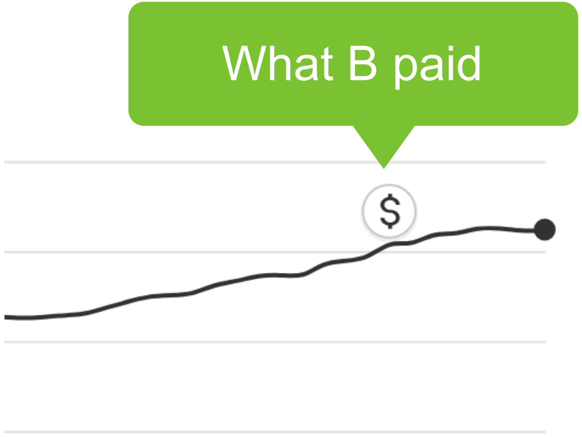
Bedroom Information
- # of Bedrooms (Minimum): 7
- # of Bedrooms (Maximum): 7

- Kitchen Description: Countertop Dishwasher, Garbage Disposal, Ho Island with Sink, Microwave, Over

House Price Prediction

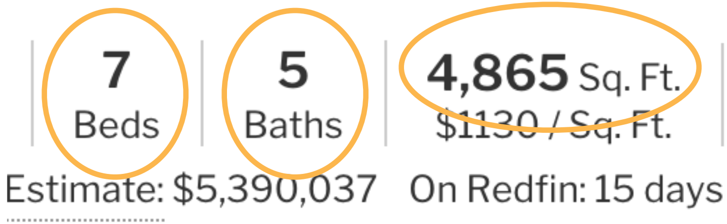
Very important, that’s real money…

$100K+ gap



Redfin overestimated the price, and B believed it

A Simplified Model



- Assumption 1
  The key factors impacting the prices are
  #Beds, #Baths, Living Sqft, denoted by x_1, x_2, x_3
- Assumption 2
  The sale price is a weighted sum over the key factors

$$y = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

Weights and bias are determined later

Linear Model

- Given *n*-dimensional inputs $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$
- Linear model has a *n*-dimensional weight and a bias

$$\mathbf{w} = [w_1, w_2, \ldots, w_n]^T, \quad b$$

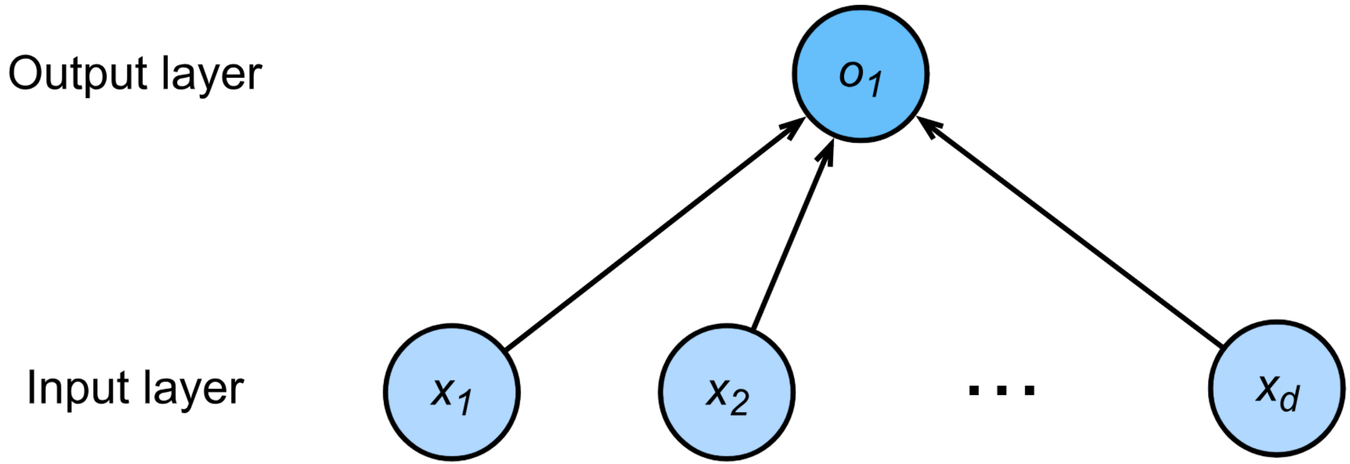
- The output is a weighted sum of the inputs

$$y = w_1 x_1 + w_2 x_2 + \ldots + w_n x_n + b$$

Vectorized version

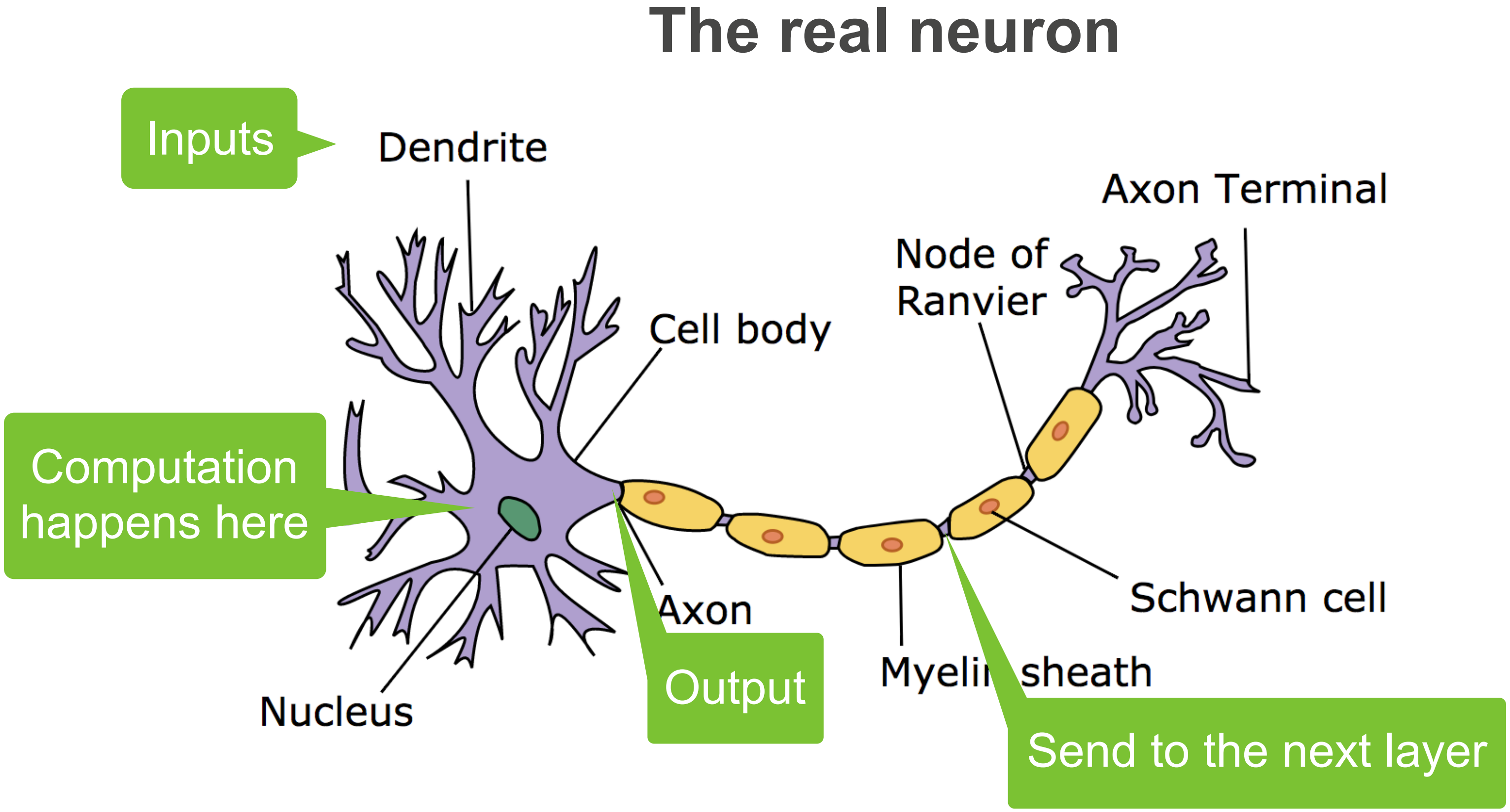
$$y = \langle \mathbf{w}, \mathbf{x} \rangle + b$$

Linear Model as a Single-layer Neural Network



We can stack multiple layers to get deep neural networks

Neural Networks Derive from Neuroscience



Measure Estimation Quality

- Compare the true value vs the estimated value
  Real sale price vs estimated house price
- Let y the true value, and $\hat{y}$ the estimated value, we can compare the loss

$$\ell(y, \hat{y}) = (y - \hat{y})^2$$

It is called squared loss

Training Data

- Collect multiple data points to fit parameters
  Houses sold in the last 6 months
- It is called the training data
- The more the better
- Assume *n* examples

$$\mathbf{X} = \left[\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n\right]^T \quad \mathbf{y} = \left[y_0, y_1, \ldots, y_n\right]^T$$

Learn Parameters

- Training loss

$$\ell(\mathbf{X}, \mathbf{y}, \mathbf{w}, b) = \frac{1}{n} \sum_{i=1}^{n} \left(y_i - \langle \mathbf{x}_i, \mathbf{w} \rangle - b\right)^2 = \frac{1}{n} \left\| \mathbf{y} - \mathbf{X}\mathbf{w} - b \right\|^2$$

- Minimize loss to learn parameters

$$\mathbf{w}^*, \mathbf{b}^* = \arg\min_{\mathbf{w}, b} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}, b)$$

Closed-form Solution

- Add bias into weights by $\mathbf{X} \leftarrow [\mathbf{X}, \mathbf{1}] \quad \mathbf{w} \leftarrow \begin{bmatrix} \mathbf{w} \\ b \end{bmatrix}$

$$\ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = \frac{1}{n} \| \mathbf{y} - \mathbf{X}\mathbf{w} \|^2 \qquad \frac{\partial}{\partial \mathbf{w}} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = \frac{2}{n} \left(\mathbf{y} - \mathbf{X}\mathbf{w} \right)^T \mathbf{X}$$

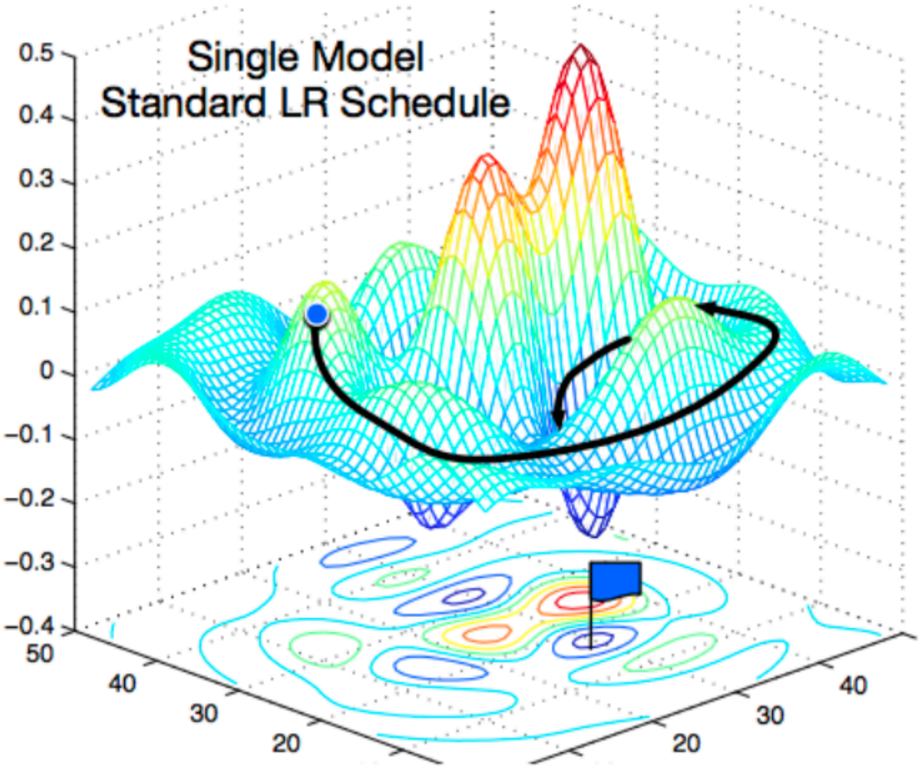
- Loss is convex, so the optimal solutions satisfies

$$\frac{\partial}{\partial \mathbf{w}} \ell(\mathbf{X}, \mathbf{y}, \mathbf{w}) = 0$$

$$\Leftrightarrow \frac{2}{n} \left(\mathbf{y} - \mathbf{X}\mathbf{w} \right)^T \mathbf{X} = 0$$

$$\Leftrightarrow \mathbf{w}^* = \left(\mathbf{X}^T \mathbf{X} \right)^{-1} \mathbf{X}\mathbf{y}$$

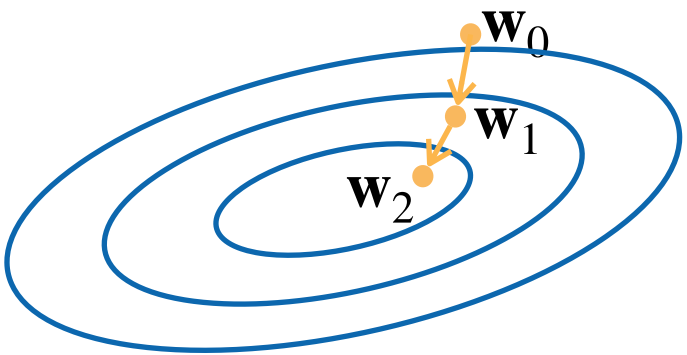
Basic Optimization



Gradient Descent

- Choose a staring point $\mathbf{w}_0$
- Repeat to update the weight *t=1,2,3*

$$\mathbf{w}_t = \mathbf{w}_{t-1} - \eta \frac{\partial \ell}{\partial \mathbf{w}_{t-1}}$$

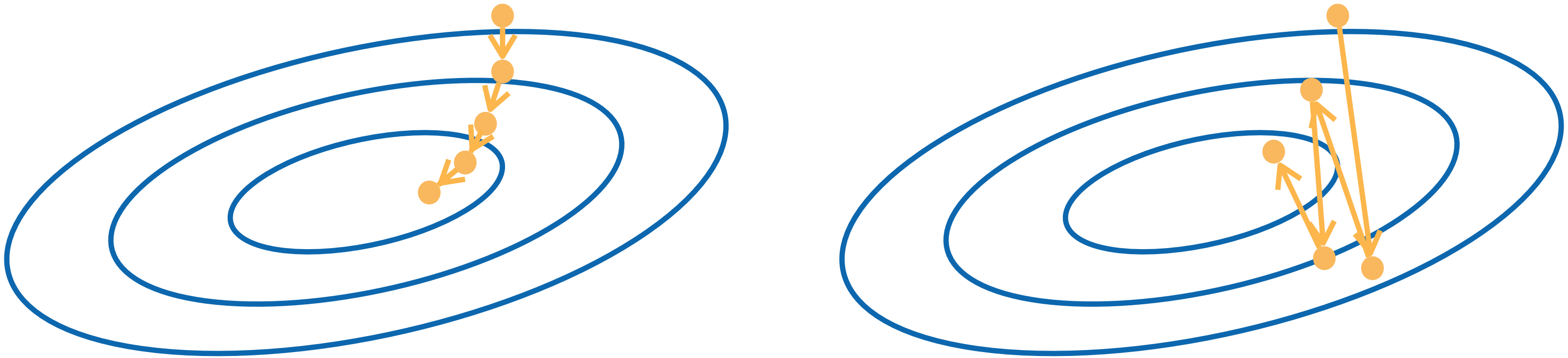


- Gradient: a direction that increases the value
- Learning rate: a hyper-parameter specifies the step length

Choose a Learning Rate

Not too small

Not too big



Mini-batch Stochastic Gradient Descent (SGD)

- Computing the gradient over the whole training data is too expensive
  - Takes minutes to hours for DNN models
- Randomly sample *b* examples $i_1, i_2, \ldots, i_b$ to approximate the loss

$$\frac{1}{b} \sum_{i \in I_b} \ell(\mathbf{x}_i, y_i, \mathbf{w})$$

  - *b* is the batch size, another important hyper-parameters

Choose a Batch Size

Not too small

Workload is too small, hard to fully utilize computation resources

Not too big

Memory issue
Waste computation, e.g. when all x_i are identical

Summary

- Problem: estimate a real value
- Model: $y = \langle \mathbf{w}, \mathbf{x} \rangle + b$
- Loss: square loss $\ell(y, \hat{y}) = \left(y - \hat{y}\right)^2$
- Learn by mini-batch SGD
    - Choose a starting point
    - Repeat
        - Compute gradient
        - Update parameters